Universidad Nacional de La Plata

Departamento de Economía
Facultad de Ciencias Económicas
Universidad Nacional de La Plata

A Note On Optimal Insurance in an Information Constrained Federal Economy with Incomplete Degree of Enforceability and Negotiation Costs

Emilio Espino

Documento de Trabajo Nro. 18
Noviembre 1999

A Note On Optimal Insurance in an Information Constrained Federal Economy with Incomplete Degree of Enforceability and Negotiation Costs*

Emilio Espino

Cornell University

Department of Economics

and

Universidad Nacional de La Plata, Argentina

Fac. de Cs. Economicas, Departamento de Economia

This Version: October 1999

Abstract

This paper studies the optimal insurance contract between a state and the central government in a federal economy with moral hazard, risk of repudiation (given some enforceability technology) and aggregate uncertainty. Also, it considers date 0 negotiation costs to implement this contract. The distribution of the fiscal resources locally collected by the province at $t+1$ are affected by period t state's effort to collect taxes. Also, every period a state has the right to get a fixed proportion of the taxes nationally collected by the central government. These resources are identically and independently distributed across time. Using a recursive formulation of the allocation problem (following Atkeson (1991)), some basic properties of the optimal insurance contract are discussed showing when, in particular, it is actually optimal just to give up any attempt to provide insurance to the province.

Under Revision-Comments Welcome

* I thank Alberto Porto for introducing me in this topic, for his patience and helpful comments and Pablo Beker for his very useful comments and critiques. The author received financial support from CONICET (Argentina) which is much appreciated. All errors are mine.

A Note On Optimal Insurance in an Information Constrained Federal Economy with Incomplete Degree of Enforceability and Negotiation Costs

1 Introduction

For different reasons, it is sometimes claimed that the optimal transfer mechanism embodied in the federal system should be such that the state or province or region can get some insurance from the central government or authority. For example, the Maastricht Treaty for European Monetary Union (EMU) entails a huge loss of monetary policy autonomy for individual countries and only limited scope for borrowing against regional shocks. Some observers have claimed that a successful EMU requires some risk sharing scheme at the community level (Persson and Tabellini [6]). In federal countries, moreover, there is some evidence that the federal system has been trying to fulfill this purpose at least partially. For the United States of America, Asdrubali et al. [1] presents evidence to quantify the amount of risk sharing across states finding that 13 percent of the shocks to gross state product are smoothed by the federal government[1] . On the other hand, in Europe the federal system provides virtually no risk-sharing. Del Negro [4] checks that these results are robust with respect to changes in both the data and the methodology.

In some Latin American countries this issue is even more complex since the federal system is also required to provide incentives to the states to create strong institutions to collect more local taxes. This is mainly because regional governments finance a high proportion of their spending with resources transferred from the central government. As a matter of fact, public

[1] They actually descompose the federal government smoothing. They find that federal grants to states smooth 2.5 percent of shocks to gross state product, which is small compared with the smoothing through the federal tax-transfer system, and that unemployment insurance smoothes 1.7 percent.

sector decentralization is under discussion to determine the optimal degree of this kind of institutional reform[2] which makes the provision of those incentives a natural initial step.

Mainly for technological reasons, in a federal economy central governments use to collect taxes all over the country and then , given some previously de…ned property rights, each state or province gets its part. States also collect their own taxes. In this situation, it is important to determine which is the optimal mechanism through which …scal resources are given back from the central government to the states. To do this, we need to discuss and justify how the information is shared all over the economy and which are all the technological aspects involved in the contract. Since these …scal resources are in general random, under the assumption of perfect information, the state should be able of smoothing its consumption of the public good getting complete insurance from the CG or, more in general, international competitive markets.

Now a state has information that is hardly observable for the central government. In particular, the e¤ort level made by the state to locally collect taxes is part of it and it is crucial to determine to optimal transfer mechanism. Also, the degree of enforceability of any contract signed by the CG and a state is actually incomplete: it is hard to imagine that a determined state will remain within the contract no matter what.

However, the characterization of the optimal transfer mechanism which provides insurance might be obscured by some facts. In particular, the empirical evidence shows that this mechanism has also been used to redistribute resources through regions, which is clearly

[2] Some evidence shows that Latin American countries had a more decentralized …scal structure in 1995 than in they had had in 1985. See Sanguinetti and Tomassi [8] for a discusion of this issue and references.

a very di¤erent objective from that concerning with optimally giving back resources in a federal economy to provide some kind of insurance. Even though redistribution aspects involved in this kind of contracts are very important[3] , this paper will purposely abstract from redistributive issues to concentrate on the risk sharing aspects.

Following Atkeson [2] in his seminal paper on international borrowing and lending, we will use some recent results on e¢cient allocations in information constrained economies with incomplete degree of enforceability to analyze the optimal properties the best mechanism should have when date 0 negotiation costs are present. Thus it will be examined the constrained optimal insurance contract from the central government to the state in an environment in which there are mainly three impediments to forming contracts. The …rst one arises from the assumption that the CG cannot observe whether states invest or consume its funds. Di¤erent levels of state's public investment will generate di¤erent probability distributions for the next period's locally collected taxes. Here, public investment should be interpreted very generally (as, for example, Persson and Tabellini [6,7]): it is re‡ecting resources devoted to improve the technology concerning the collection of taxes in a given state. This assumption leads to a moral hazard problem in investment. The second impediment arises from the assumption that the state ("the borrower"), as a relatively free state in a federal economy, may choose to repudiate his debt ("risk of repudiation") at some cost given that the enforceability technology available in this economy is not perfect; this is what we call incomplete degree of enforceability. Finally, the implementation of any insurance contract di¤erent from autarky is costly, representing the fact that new institutions must be

[3] For a recent discussion of this issue, see Persson and Tabellini [7].

created.

This note will then proceed with the following logic. First we will specify the characteristics the optimal contract should have if full observability, complete enforceability and costless contracts were feasible. Then, once the main aspects involved in the relationship between the CG and the state have been discussed, it is very important from the theoretical point of view, and as a first step, to specify the objective and the constraints the problem-solver has to face. Thus, when the relevant allocation problem has been stated, it will be analyzed the optimal contract between the central government and the state and its properties, whenever it is possible to characterize them. It will be shown that there are cases where no insurance is actually optimal when the negotiation process is considered. Roughly speaking, when constructing the optimal transfer mechanism, the central government needs to generate incentives to have the state make determined levels of effort (given the aggregate state in the economy reflected for both income shocks). It may be the case that when a low level of locally collected resources is observed, and independently of the realization of nationally collected taxes, the payoff for the state when remaining within the contract is low to provide these incentives and so, when negotiated at date 0, the state would prefer not to get any insurance from the central government just to avoid "paying" that ex-ante insurance cost. Therefore, certain specific bounds to these costs will be found for same cases. We will also establish informally why it seems possible situations where there are incentives to renegotiate.

These arguments could then be useful to justify the fact that maybe the best institutional and political solution is just to give back the …scal resources without trying to smooth state's consumption, and so the state will solve its "autarky" problem. There, a state takes its …scal resources and maximizes some objective function. Motivated by plausibility and simplicity, we will assume that the state has no access to any additional credit market.

It should be emphasized that this note is a theoretical exercise formalizing simple ideas. It attempts to be useful in the policy analysis to get insights about some problems arising when a federal system is trying to be implemented.

Before presenting the model economy, some related literature must be mentioned. In a static set up, Persson and Tabellini [6] studies policy outcomes under di¤erent federal institutional arrangements (vertically and horinzontally ordered) where collectively chosen …scal policy shares risks between individuals and between regions, and alters the probability distribution of aggregate shocks[4] . Ex ante public investment is unobservable and it can positively a¤ect the distribution of regional income (investing more resources makes the good state more likely). Agents are arbitrarily not able of getting insurance with respect to their types (idiosyncratic probabilities of getting a job o¤er). However, regional governments can implement an unemployment insurance program which is chosen through majority rule and,

[4] See also their companion paper, Persson and Tabellini [7] where they study the political and economic determinants of interregional public transfers. It focuses on how much transfers are shaped by alternative …scal constitutions, where a constitution is an allocation of …scal instruments across di¤erent levels of governments plus a procedure for the collective choice of these …scal instruments. Thus restrictions on …scal instruments introduce a trade-o¤ between risk sharing and redistribution where di¤erent constitutions produce di¤erent results. A federal social insurance scheme, chosen by voting, provides overinsurance, whereas an intergovernmental transfer scheme, chosen by bargaining, provides underinsurance. As they specify, this analysis is purely positive since they arbitrarily impose the relevant instruments. The present paper is however purely normative since we will analyze the properties of the optimal transfer mechanism, given preferences, technologies and property rights but it does not discuss issues related to redistribution across states(in particular, it takes as given the proportion of …scal resources coming from nationally collected taxes corresponding to the state).

given their assumptions, it is found optimal to support full social insurance. When analyzing risk sharing across regions, two alternative institutional arrangements are related to this paper. In one of them, regional policies (the unemployment insurance program and the level of public regional investment) and the national policies (the aggregate risk-sharing contract across regions) are voted simultaneously. Here, aggregate risk sharing can exacerbate the moral hazard problem: the more regions are insured, the smaller their incentive to invest. As in the present paper, there is thus a trade-o¤ between risk sharing and moral hazard In the second arrangement, the federal policy is voted prior to the regional policy. Here, the resulting political equilibrium entails a more favorable trade-o¤ between risk sharing and moral hazard where a second best allocation can be implemented. In our paper, the contract considered to share risk is the optimal one with no additional restrictions imposed in advanced (for example, transfers between the central government and the state will be contingent to the aggregate state of the economy; this is not the case in their paper). Also, in the present paper we will additionally consider a dynamic environment with incomplete degree of enforceability.

Sanguinetti and Tomassi [8] studies two alternative regimes where idiosyncratic shocks to regions are private information. In one regime, the central government commits to a certain level of transfers to compensate vertical …scal imbalances and so provide some limited ex-ante insurance (by increasing the expected value of local consumption, which is subject to stochastic shocks). In the other regime, it accommodates ex-post the …scal needs of the di¤erent provinces, after the local government has made its choices. In this second case, full-insurance results, but the economy is subject to a tragedy of the fiscal commons, with excessive regional spending and reduced production of federal public goods (such as macroeconomic stability).

This paper is organized as follows. Section 2 describes the economy, specifies basic definitions and discuss the definition of a constrained efficient allocation. It also includes a characterization of the costless, fully observable and completely enforceable contract if the state has access to perfect international credit markets. Section 3 discusses a recursive formulation of this problem and presents some basic properties of the optimal contract discussing situations where no insurance is actually an efficient outcome. Section 4 concludes.

2 The Model Economy

As mentioned in the introduction, we use a modified version of the model introduced by Atkenson [2]. There is a state representing an infinitely-lived, risk-averse agent, whom we call simply the state (S)[5] . The state's consumption of the public good in period t is denoted g_t. In addition, there is an infinitely-lived risk-neutral central government, denoted CG. Time is discrete and denoted by $t = 0; 1; 2; :::$.The state has a stochastic technology to locally collect taxes. Increased investment by the state makes good realizations more likely. We interpret this such that increased "effort" by the state (represented by investment in terms of the public good) increases the probabilities of getting large amounts of state's fiscal resources. Moral hazard will constrain contracts between the state and CG because it is

[5] The local government's preferences, in trivial application of the median voter theorem, coincide with that of the representative agent. However, the agent not necessarily consumes only this public good; they may have been chosen their optimal contingent bundles of private goods given $\{g_t\}$ and some taxation policy. The strong assumption will appear when imposing conditions to the shape of $u(.)$.

assumed that the state's consumption of the single (public) good and the level of its (public) investment are indeed unobservable. So given state's investment in units of goods at time t (I_t), the state's investment opportunity yields as next period state's fiscal resources the random variable Y_{t+1} which takes values in the finite set $\Omega = (Y_1; \ldots; Y_N)$; where $Y_i > Y_j$ if $i > j$: Denote $f(Y_{t+1}; I_t)$ the probability distribution of Y_{t+1} given I_t and assume that f is continuous with respect to I_t (and so measurable):We assume that $f(Y_{t+1}; I_t) > 0$ such that the CG cannot infer any state's effort level from the realization of Y. Note that we can interpret that Y_{t+1} is just a fixed proportion of the state's total output when the state's taxation policy is taken as given. Note also that we are assuming that this fiscal income is not depending on any central government's activity.

The CG is endowed with a random large quantity of fiscal resources $\{\theta_t\}$ of the same good which is identically and independently distributed through time and independent with respect to Y_t: We assume that θ_t takes values in the finite set $\Theta = \{\theta_1; \ldots; \theta_J\}$; with probability distribution π and $\theta_i > \theta_j$ if $i > j$. Let $\overline{\theta} = \sum_{j=1}^{J} \pi(\theta_j)\theta_j$: Assume that a fixed fraction α of θ_t is transferred every period to the state. The decision of this last proportion is assumed to have been decided in some previous (political) process and it is taken as given. Note that this (random) income for the state is representing aggregate uncertainty.

We now separate the total amount of transferred funds to the state in period t in two contingent components: one representing the repayment from the state to the CG given that the state has received a transfer from the CG in period $t - 1$, represented by d_t; the second one representing a new gross transfer, represented by b_t. Thus, the net transfer to the state in period t is given by $b_t - d_t$, which is representing the negative CG's consumption net of endowment $(1-\alpha)\theta_t$. That is, CG's consumption at date t is given by $(1-\alpha)\theta_t - b_t + d_t$; which must be nonnegative.

Given that $\{\theta_t\}$ is assumed to be exogenously given, an allocation in this environment is defined to be a plan which specifies current consumption of the state, current net transferred funds and investment in the tax-collecting technology. Given that $\{g_t; I_t\}$ are unobservable, the plan may depend on the entire observable history of realization of fiscal resources. Hence, we choose $Q_t = Y_t - d_t$ and θ_t as the state variables and write $x^t = (x_0; \ldots; x_t)$ to represent the partial history up to date t. Thus, an allocation is represented by the stochastic process

$$\sigma = \{g_t(Q^t;\theta^t); b_t(Q^t;\theta^t); I_t(Q^t;\theta^t); d_{t+1}(Y_{t+1};\theta_{t+1};Q^t;\theta^t)\}_{t=0}^{\infty}$$

with initial conditions Y_0; θ_0 and d_0. As usual we will assume that $d_0 = 0$ (i.e. the state starts negotiating with CG about the optimal transfer mechanism having no initial debt).

Definition 1 An allocation is feasible if for all t; for all $(Y_t;\theta_t)$ and for all $(Y^t;\theta^t)$

$$g_t(Q^t;\theta^t) - [b_t(Q^t;\theta^t) - d_t(Y_t;\theta_t;Q^{t-1};\theta^{t-1})] + I_t(Q^t;\theta^t) \le Y_t + \alpha\theta_t \tag{1}$$

where $g_t(Q^t;\theta^t); I_t(Q^t;\theta^t) \geq 0$
and $b_t(Q^t;\theta^t) - d_t(Y_t;\theta_t;Q^{t-1};\theta^{t-1}) \le (1-\alpha)\theta_t$

Let us impose two additional assumptions about the CG. First, the CG binds itself to carry out the terms of a contract only it is getting, in expected value, at least 0 in every two-period contracts. We can interpret this assumption just like being an institutional rule imposed to the CG through some previous political decision[6]. Second, we assume that when

[6] We impose this assumption to reflect the fact that the CG is more impatient than that represented by the discount factor.

the CG has suffered a repudiation to its transfer contract it may costlessly seize any positive transfer the state might have to receive in the future as compensation towards its loss and also it can take away a technologically given fraction (1 $-$) of the nationally collected fiscal resource corresponding to the state at that date. The parameter represents one side of the degree of enforceability in this economy.

Preferences

State's preferences over allocations are represented by a utility function denoted by

$$U^S(\sigma) = (1-\beta)E_0^\sigma\left\{\sum_{t=0}^{\infty}\beta^t u(g_t(Q^t;\theta^t))\right\}$$

That is, the household represented by the state has preferences represented by a time separable, expected, discounted utility function (where $\beta \in (0;1)$). We assume that the momentary utility function satisfies: (i) u is bounded above by $\bar{u}$; (ii) $u' > 0; u'(0) = \infty$ and $u'' < 0$. E_0^σ denotes the conditional expectation on the information available at time 0[7].

The CG has preferences represented by the expected discounted value of its consumption

$$\begin{aligned} U^{CG}(\sigma) &= E_0^\sigma\left\{\sum_{t=0}^{\infty}\beta^t[(1-\alpha)\theta_t - b_t(Q^t;\theta^t) + d_t(Y_t;\theta_t;Q^{t-1};\theta^{t-1})]\right\} \\ &= \frac{(1-\alpha)\bar{\theta}}{(1-\beta)} - E_0^\sigma\left\{\sum_{t=0}^{\infty}\beta^t[b_t(Q^t;\theta^t) - d_t(Y_t;\theta_t;Q^{t-1};\theta^{t-1})]\right\} \end{aligned}$$

Therefore, since affine transformations of $U^{CG}(\sigma)$ will represent the same CG's preference

[7] Expectations are taken with respect to the probability measure induced by the allocation and the given probability distribution π: Note that, given an allocation, $I_t(Q^t;\theta^t)$ and π determine a unique probability measure μ on the σ-field generated by the stochastic process $\{Q_t;\theta_t\}$:

ordering, we can write

$$U^{CG}(\sigma) = -E_0^{\sigma}\left\{\sum_{t=0}^{\infty}\beta^t[b_t(Q^t;\theta^t) - d_t(Y_t;\theta_t;Q^{t-1};\theta^{t-1})]\right\}$$

We confine ourselves to examining allocations which are both feasible and which provide both governments with at least as much utility as could be obtained by not negotiating at all, since participation in the contract is voluntary. The reservation utility of the CG in this environment is zero. The reservation utility of the state is the expected utility it can get refusing all kind of transfers and consuming and investing in the storage technology on its own while every period the state gets from the central government the random income $\alpha\theta_t$. Thus, this can be gotten as the solution to the following dynamic programming problem[8] :

$$U^S_{aut}(Y;\theta) = \max_{I\in[0;Y+\alpha\theta]}\left\{(1-\beta)u(Y+\alpha\theta - I) + \beta\sum_{Y';\theta'}U^S_{aut}(Y';\theta')f(Y';I)\pi(\theta')\right\}$$

Note that, since u is strictly increasing, it follows by standard arguments U^S_{aut} is strictly increasing in both arguments.

Assumption 1:

$$(1-\beta)u(0) + \beta\bar{u} < U^{aut}(Y_1;\theta_1)$$

This condition ensures that there are levels of current consumption so low that the state prefers the autarky allocation to an allocation which specifies these low levels of current consumption, regardless of what levels of consumption were to be offered in the future by the contract.

[8] Note that this dynamic programming problem has a well-defined solution; see Stokey, Lucas with Prescott [9].

Assumption 2[9] : The distribution of the state's fiscal resources given investment, $f(Y,I)$, is given by the convex combination of two underlying distributions $f_0(Y)$ and $f_1(Y)$ as follows:

$$f(Y,I) = \lambda(I)f_0(Y) + (1-\lambda(I))f_1(Y)$$

with $f_0(Y_i)/f_1(Y_i)$ monotone increasing in i, $\lambda(I) \in [0,1]$, $\lambda'(I) > 0$; and $\lambda''(I) \leq 0$ for all I. Note $\frac{\partial f(Y,I)}{\partial I} = f_I(Y,I) = \lambda'(I)[f_0(Y) - f_1(Y)]$.

Note that after (Q^t,θ^t) has been realized, the continuation value of a given allocation is represented by

$$U^S(\sigma/Q^t,\theta^t) = (1-\beta)u(g_t(Q^t,\theta^t)) + E_t^\sigma\left\{\sum_{s=1}^{\infty}\beta^{t+s}u(g_{t+s}(Q^{t+s},\theta^{t+s}))/Q^t,\theta^t\right\}$$

where as usual $E_t^\sigma\{\ldots/Q^t,\theta^t\}$ is the conditional expectation (given the σ-algebra generated by (Q^t,θ^t)[10]).

Negotiation Costs

We introduce the assumption that the negotiation process has a cost in terms of state's current fiscal income in period 0. Suppose that this is represented by

$$\omega_0(\sigma) = \begin{cases} 0 & \text{if } \sigma \text{ is the autarky allocation} \\ \omega > 0 & \text{otherwise} \end{cases}$$

This is representing the fact that negotiating something different from just the devolution of the state's fiscal resources has additional costs (which may be very large). We should interpret these costs very generally: they are representing the costs of creating all the new

9 This assumption justifies the use of the first order conditions.

10 See Billingsley [3], chapter 34, for details.

institutions to implement the optimal contract (to provide, for example, an additional degree of commitment). Note that the definition of feasibility in period 0 depends upon which is the relation between the CG and the state. This dependence is obvious. Note also that negotiation costs are paid just in period 0.

We will assume that the CG will provide as much insurance to the state as possible but, when considering the date 0 participation constraint, it will not try to provide expected utility such that the state is better off within the contract after paying these negotiation costs by itself. CG will just provide expected utility such that the state is better than the autarky level, without considering these additional costs.

Definition 2 A feasible allocation σ is **individually rational** if for all t and for all $(Q^t;\theta^t)$

$$(2)\qquad U^S(\sigma|Q^t;\theta^t)\ \geq\ U^S_{aut}(Q_t;\theta_t)\ \text{and}\ U^{CG}(\sigma)\ \geq\ 0$$

Moreover, a feasible allocation σ satisfies the **institutional rule** if for all t and for all $(Q^t;\theta^t)$

$$(2')\qquad -b_t(Q^t;\theta^t)+\beta\sum_{Q';\theta'} d_{t+1}(Y';\theta';Q^t;\theta^t)f(Y';I_t(Q^t;\theta^t)\pi(\theta')\ \geq\ 0$$

Note that the individual rationality constraint for the CG holds when the institutional rule is satisfied . To see this, note that

$$\begin{aligned}
&-b_t(Q^t;\theta^t)+\beta\sum_{Q';\theta'} d_{t+1}(Y';\theta';Q^t;\theta^t)f(Y';I_t(Q^t;\theta^t))\pi(\theta')\\
=\ & E^{\sigma}_t\{-b_t+\beta d_{t+1}|(Q^t;\theta^t)\}
\end{aligned}$$

almost surely since $d_{t+1}(Y';\theta';Q^t;\theta^t)f(Y';I_t(Q^t;\theta^t))$ and $b_t(Q^t;\theta^t)$ are $(Q^t;\theta^t)$-measurable and applying conditional expectation to the second term. Thus, by definition of conditional expectation,

$$E^{\sigma}_0\{-b_t(Q^t;\theta^t)+\beta\sum_{Q';\theta'} d_{t+1}(Y';\theta';Q^t;\theta^t)f(Y';I_t(Q^t;\theta^t))\pi(\theta')\}$$

$$= E_0^{\sigma}\{-b_t + \beta d_{t+1}\}$$

Therefore, if the institutional constraint is satisfied, since previous equality holds for every t; reordering in the obvious way the individual rationality constraint for the CG, we get that the last one will hold.

2.1 Costless Efficient Contracts with Perfect Enforceability and Full Observability

Before analyzing the constrained efficient contract (given the problems of moral hazard and the risk of repudiation generated by the incomplete degree of enforceability), we will discuss the contract emerging if costless, perfectly enforceable and fully observable contracts were feasible.

Suppose then that the state can issue Arrow securities and that there is a large number of risk-neutral lenders. Let $a(Y';\theta')$ be the number of Arrow securities issued by the sate for the next period's realization $(Y';\theta')$; let $p(Y';\theta')$ be the price of this security. With a large number of risk-neutral lender, it is well-known that $p(Y';\theta') = \beta\pi(\theta')f_I(Y';I)$ given some level of state's investment ("effort") I. Let $v(w)$ be the state's value function given a level of state's current fiscal resources equal to w. It can be shown by standard arguments that $v(w)$ is bounded, strictly increasing, strictly concave and continuously differentiable. Note that given $(Y;\theta)$ we have that $w = Y + a(Y;\theta) + \alpha\theta$: The state's allocation problem can be shown to have the following Bellman equation representation:

$$v(w) = \max_{I;\{a(Y';\theta')\}} \{(1-\beta)u(w - \sum_{(Y';\theta')} p(Y';\theta')a(Y';\theta') - I) + \beta \sum_{Y';\theta'} v(w')f(Y';I)\pi(\theta')\}$$

subject to $[w - \sum_{(Y';\theta')} p(Y';\theta')a(Y';\theta') - I] \geq 0$ and $I \geq 0$; where $w' = Y' + a(Y';\theta') + \alpha\theta'$

First order conditions for interior solutions are given by

$$a(Y';\theta') \quad : \quad (1-\beta)u'(c)p(Y';\theta') = \beta v'(Y' + a(Y';\theta') + \alpha\theta')f(Y';I)\pi(\theta')$$

$$I \quad : \quad (1-\beta)u'(c) = \beta \sum_{Y';\theta'} v(Y' + a(Y';\theta') + \alpha\theta')f_I(Y';I)\pi(\theta')$$

The envelope condition is given by

$$v'(w) = (1-\beta)u'(c)$$

Notice that from the first F.O.C., the asset pricing formula previously described and the envelope condition, we get that

$$u'(c) = u'(c')$$

and so $c = c'$. Therefore consumption is completely smoothed through time and different realizations of $(Y';\theta')$. But then since v is strictly decreasing, it follows form the envelope condition and the fact that consumption is completely smoothed that $Y + a(Y;\theta) + \alpha\theta = Y' + a(Y';\theta') + \alpha\theta' = \overline{w}$ for all $(Y;\theta)$ and all $(Y';\theta')$. Thus $a(Y';\theta') = \overline{w} - Y' - \alpha\theta'$ for all $(Y';\theta')$. Finally, notice that investment is also constant through time and different realizations of $(Y';\theta')$. Note how this affects the determination of the Arrow securities' prices.

This outcome with full insurance is the usual solution with complete observability and perfect enforceability. Now we will consider the case where these two properties are missed.

2.2 The Constrained Efficient Contract

We define the constraints on the set of allocations that are imposed by the problems of moral hazard and the risk of repudiation generated by the incomplete degree of enforceability. Then we set up the problem of finding the constrained efficient transfers.

To define the set of allocations which are free from the risk of repudiation, we must describe the set of punishments that the CG can impose upon the state for repudiation. CG can punish the state for repudiation by refusing it further positive transfers and also by taking away an additional fraction $(1 - \;)$ of the nationally collected resources at date t. The technological parameter , taken as given, is representing the fact that the degree of enforceability is incomplete in this economy[11] . Thus if the state finds optimal to repudiate any negative transfer at date t, the worst punishment that the CG can impose upon the state who repudiates a negative transfer at some period is the state's autarky utility after having substructed $(1 - \;)$ from the nationally collected taxes going to the state[12] .

Definition 3 An allocation σ is **immune from the threat of repudiation** if for all $t \geq 0$ and for all Y' and θ', the continuation allocation after the realizations of fiscal resources from the storage technology and those nationally collected, satisfies:

$$U^S(\sigma / Q^t; \theta^t; Y'; \theta') \geq U^S_{aut}(Y'; \theta') \tag{3}$$

An allocation σ is **incentive compatible** if for all feasible allocation $\hat{\sigma} = \{\hat{g}_t(); \hat{b}_t(); b_t(); d_{t+1}()\}$ (with the components $\{b_t\}$ and $\{d_{t+1}\}$ unchanged):

$$U^S(\sigma) \geq U^S(\sigma') \tag{4}$$

Condition (3) is the ex-post participation constraint; thus the state must find optimal to continue within the contract next period when considering $U^S_{aut}(Y'; \theta')$ as its ex-post reservation value. The relevant reservation value for the CG is that imposed by the institutional rule discussed above. We will then examine self-enforcing contracts where neither party ever has an incentive to renege.

[11] As it will be discussed below, it is very important that is taken as given and not decided in some political process at date 0.

[12] This concept is also used in Espino (1999) in a closely related economy.

Condition (4) says that an allocation is incentive compatible if the state gets more utility from carrying out the contingent consumption and investment plans speci…ed in the allocation than from considering any other consumption and investment plan whenever it takes the lending and repayment plans speci…ed as given.

Now we are ready to state the insurance problem the CG will solve. We will assume the extreme case where it will provide as much insurance to the state as possible without taking into account date 0 negotiation costs paid by the state.

Note that we assuming away the possibility that the CG repudiates any contract.

De…nition 4 An allocation ¾¤ is **Constrained E¢cient** if it maximizes U^S(¾) subject to the constraints of (1) feasibility, (2) individual rationality and the institutional rule, (3) immunity from the threat of repudiation, and (4) incentive compatibility.

The di¢cult part of solving this program is understanding how to handle the incentive compatibility constraint. If positive level investment is speci…ed in an allocation with full insurance, it will not be incentive compatible: as we have shown before, state's level of …scal resource would be constant and since investment is unobservable the state will invest nothing (i.e. it will make no "collecting e¤ort").

We will proceed as follows. First we solve the programming problem assuming no negotiation costs and then we will analyze the problem in the …rst period (where these costs are assumed to be "paid" by the state). Thus, we do not consider the date 0 participation constraint for the state taking into account negotiation costs What we are doing is just computing the expected utility the state would get following the optimal partial insurance contract and then check the participation constraint for period 0. Note that, even though when computing σ^* we will have that $U^S(\sigma^*) \geq U^S_{aut}(Y_0;\theta_0)$, it does not necessarily follow that $U^S(\sigma^*) \geq U^S_{aut}(Y_0 - \omega;\theta_0)$: In that case, σ^* is actually σ_{aut}:

3 Recursive Formulation and Some Properties of the Constrained Efficient Contract

Define, for each value of $(Q;\theta)$, the state's utility possibility correspondence (that is, the levels of state's expect utility that can be achieved when moral hazard and incomplete enforceability are taking into account) as

$$V(Q;\theta) = \{U^S(\sigma) : \sigma \text{ satisfies (1)-(4) and } (Q_0;\theta_0) = (Q;\theta)\}$$

The correspondence V is not empty valued since $U^S_{aut}(Q;\theta) \in V(Q;\theta)$ for all $(Q;\theta)$ and bounded (from below by 0 and from above by $\overline{U} < \infty$). Now, let W be any correspondence defined over domain $Q \times \Theta$, with $W(Q;\theta)$ non-empty and uniformly bounded for all values in the domain. Define a set of current controls to be the vector $A = (g; l; b; d')$ where g, b and l are scalars and $d' : Y \times \Theta \to R$. Define a function U to be a continuation value function if it is a selection from the correspondence W; i.e. $U : Q \times \Theta \to R$, with $U(Q;\theta) \in W(Q;\theta)$ for all values of $(Q;\theta)$.

Definition 5 The pair $(A;U)$ of current controls and continuation value function, is admissible with respect to W at $(Q;\theta)$ if it satisfies the following conditions:

$$(1') \qquad g - b + l \leq Q + \alpha\theta$$

where $g; l \geq 0$ and $b - d \leq (1 - \alpha)\theta$

$(2')$

$$(1-\beta)u(Q+\alpha\theta+b-l)+\beta\sum_{Y',\theta'}U(Y'-d'(Y',\theta'),\theta')f(Y',l)\pi(\theta')\geq U^S_{aut}(Q,\theta)$$

$$-b+\beta\sum_{Q',\theta'}d'(Y',\theta')f(Y',l)\pi(\theta')\geq 0$$

Also, for all (Y',θ')

$$U(Y'-d'(Y',\theta'),\theta')\geq U^S_{aut}(Y',\theta')$$

$$l\in\arg\max_{\tilde{l}}\,(1-\beta)u(Q+\alpha\theta+b-\tilde{l})+\beta\sum_{Y',\theta'}U(Y'-d'(Y',\theta'),\theta')f(Y',\tilde{l})\pi(\theta')$$

Denote the payoff to the state generated by a pair (A,U) by

$$E[(A,U)(Q,\theta)]=(1-\beta)u(Q+\alpha\theta+b-l)+\beta\sum_{Y',\theta'}U(Y'-d'(Y',\theta'),\theta')f(Y',l)\pi(\theta')$$

Denote

$$B(W)(Q,\theta)=\{E(A,U)(Q,\theta):(A,U)\text{ is admissible with respect to }W\text{ at }(Q,\theta)\}$$

Result 1: $V(Q,\theta)=B(V)(Q,\theta)$ for all (Q,θ).

Proof. It follows, after some manipulations[13], by proposition 1 and 2 given by Atkeson (1991). ■

Define the value of the optimal contract as a function of the state variables:

$$v(Q,\theta)=\sup\{v:v\in V(Q,\theta)\}$$

Hence, it follows from Result 1 that $v(Q,\theta)$ is characterized by the program:

$$\text{(P)}\qquad v(Q,\theta)=\sup_{A,U}\{(1-\beta)u(Q+\alpha\theta+b-l)+\beta\sum_{Y',\theta'}U(Y'-d'(Y',\theta'),\theta')f(Y',l)\pi(\theta')\}$$

[13] Details are available upon request to the author.

subject to the constraint that $(A; U)$ be admissible with respect to V at $(Q; \theta) = (Q_0; \theta_0)$[14].

It can be shown that V has a compact graph (and so there exists an optimal contract), that the correspondence I^* defined by (4) is single valued, that the correspondence V is continuous and that (under some additional assumptions) the function v is continuous.

Result 2: Assume that the value function v is continuous. Then the continuation value function $\hat{v}$ which solves the program (P) necessarily satisfies $\hat{v} = \bar{v}$:

Proof. See Atkeson (1991) for details. ■

The Optimal Contract

We examine some properties of the optimal contract between the CG and the state. To do this, we use the following two additional assumptions (see [2] for additional discussion).

Assumption 3: Assume that the value of repayments at the optimum is increasing in investment for all θ':

$$\sum_{Y'} d'(Y'; \theta')(f_0(Y') - f_1(Y')) \geq 0$$

This amounts to an assumption that, at the constrained optimum, the lender would prefer that the borrower make larger rather than smaller investments.

Assumption 4: Assume that the constrained optimal investment level is interior.

With these assumptions, the optimal incentive compatible level of investment I^* is the unique solution to the first order condition:

$$-(1-\beta)u'(Q+b-I) + \beta\lambda'(I) \sum_{Y';\theta'} v(Y' - d'(Y'; \theta'); \theta')(f_0(Y') - f_1(Y'))\pi(\theta) = 0$$

14 Since we are maximizing the payoff to the state, the individual rationality constraint is never binding in this program.

Given our assumptions we can replace (4) by with equality. Now, we may write our programming program in terms of controls and the continuation value function as[15] :

$$(P^{*}) \qquad v(Q;\theta) = \max_{A;U_d} \left\{(1-\beta)u(Q+\alpha\theta+b-l) + \beta \sum_{Y';\theta'} U(Y'-d(Y';\theta');\theta') f(Y';l)\pi(\theta')\right\}$$

subject to

$(1')$ $\quad g - b + l \leq Q + \alpha\theta$

where $g; l \geq 0$ and $b - d \leq (1-\alpha)\theta$

$(2')$

$$-b + \beta \sum_{Q';\theta'} d'(Y';\theta') f(Y';l)\pi(\theta') \geq 0$$

and for all $(Y';\theta)$

$(3')$ $\quad U(Y' - d(Y';\theta');\theta') \geq U^{S}_{aut}(Y';\ \theta')$

$(4')$ $\quad -(1-\beta)u'(Q+b-l) + \beta\lambda'(l) \sum_{Y';\theta'} U(Y' - d(Y';\theta');\theta')(f_0(Y') - f_1(Y'))\pi(\theta) = 0$

$(5')$ $\quad \overline{V}(Y' - d'(Y';\theta');\theta') \geq U(Y' - d(Y';\theta');\theta')$

The following result follows by standard arguments since u is strictly increasing.

Result 3: v is strictly increasing in both arguments.

Finally, we define the Lagrangian for the program (P^{*}) in terms of controls and continuation values, where the notation for the Lagrange multipliers is obvious (note that they were

[15] Note that (5') is representing admissibility of the continuation value function since v is, by definition, the supremum of V.

already normalized).

$$
\begin{aligned}
L(A;U_d;\eta) \;=\;& (1-\beta)u(Q+\alpha\theta+b-I)+\beta\sum_{Y',\theta'}U(Y'-d(Y';\theta');\theta')f(Y';I)\pi(\theta')+\\
&+\eta_1\Big[-b+\beta\sum_{Q',\theta'}d'(Y';\theta')f(Y';I)\pi(\theta')\Big]\\
&+\beta\sum_{Y',\theta'}\eta_2(Y';\theta')f(Y';I)\pi(\theta')\big[U(Y'-d(Y';\theta');\theta')-U^S_{aut}(Y';\theta')\big]\\
&+\eta_3\Big[-(1-\beta)u'(Q+b-I)-\beta\lambda'(I)\sum_{Y',\theta'}U(Y'-d(Y';\theta');\theta')(f_0(Y')-f_1(Y'))\pi(\theta)\Big]\\
&+\beta\sum_{Y',\theta'}\eta_4(Y';\theta')f(Y';I)\pi(\theta')\big[\overline{V}(Y'-d'(Y';\theta');\theta')-U(Y'-d(Y';\theta');\theta')\big]
\end{aligned}
$$

Consider the first order condition of this Lagrangian L with respect to the continuation values $U(Y'-d(Y';\theta');\theta')$; that is given by

$$
\begin{aligned}
0 \;=\;& \beta g(Y';I)\pi(\theta')+\beta\eta_2(Y';\theta')f(Y';I)\pi(\theta')\\
&+\eta_3\beta\lambda'(I)\pi(\theta')[f_0(Y')-g_1(Y')]\\
&-\beta\eta_4(Y';\theta')f(Y';I)\pi(\theta')
\end{aligned}
$$

which can be rewritten as

$$
1+\eta_3\lambda'(I)f_1(Y')[f_0(Y')/f_1(Y')-1]/f(Y';I)=\eta_4(Y';\theta')-\eta_2(Y';\theta')
$$

Since all the multipliers are nonnegative, $\eta_3 > 0$ and the fact that $f_n(Y') > 0$ for all $Y' \in \Omega$, then $\eta_2(Y';\theta') > 0$ whenever $1+\eta_3\lambda'(I)[f_0(Y')-f_1(Y')]/f(Y';I) < 0$. Thus, the no repudiation constraint binds when $\eta_3\lambda'(I)[f_0(Y')-f_1(Y')]/f(Y';I)$ is sufficiently small. Note that to get this result it is important that the state's effort (represented by investment) has an important impact on the probabilities of getting high locally collected fiscal resources.

By assumption 2, the ratio $f_0(Y')/f_1(Y')$ monotone increasing in Y', so that if it is sufficiently small for some Y', it is so for all lower realizations of Y'. However, it cannot be the case that this ratio is smaller than 1 for all Y since f_0 and f_1 are probability distributions. Is important to notice that this property does not depend upon the next period's realization θ'. That is, the difference $\delta_4(Y', \theta') - \delta_2(Y', \theta')$ is in fact independent of θ' (the left hand side in the equality above depends only upon (Q, θ) and Y').

When the no repudiation constraint binds we have that, given that $\in [0,1]$, $v(Y' - d'(Y', \theta'), \theta') = U^S_{aut}(Y', \theta') \quad U^S_{aut}(Y', \theta')$. Since the autarky allocation is always feasible, individually rational, immune to repudiation and incentive compatible, we have that $U^S_{aut}(Y' - d'(Y', \theta'), \theta') \quad U^S_{aut}(Y', \theta')$ and thus $d'(Y', \theta') \geq 0$ (with strict inequality if < 1).

Implementing the optimal contract with negotiation costs. Suppose that, a state says "yes" to the contract proposed by the CG when it is getting in period 0 at least the autarky expected utility level with local fiscal resources $Y_0 - \omega$. If the state is not satisfied with the contract, the result of the negotiation process is just the autarky allocation.

Note then that if the realization of the economy at date 0 is Y' such that the no repudiation constraint is binding, then whenever $\omega \geq d'(Y', \theta')$ we will have that for all θ $U^S_{aut}(Y', \theta) > U^S_{aut}(Y', \theta) = v(Y' - d'(Y', \theta), \theta) \geq v(Y' - \omega, \theta)$. In this situation, the solution to the problem will be "autarky". So we had established specific bounds to ω to identify situations where the solution to the negotiation process is the autarky allocation with a CG providing no insurance to a given state.

Also note that the no repudiation constraint binds, $U^S_{aut}(Y', \theta) > U^S_{aut}(Y', \theta) = v(Y' -$

$d^0(Y^0;\mu)$, what could be given a hint that there exist incentives to renegotiate. Of course, this has not been formalized here.

4 Conclusions

We have discussed in this note that in economies with asymmetric information, incomplete degree of enforceability and high negotiation costs it could be optimal to implement the autarky allocation without providing any kind of insurance from the central government to the state to avoid negotiation costs.

Basically, if state's e¤ort has an important impact on the probabilities of getting high ...scal resources, then it could be the case that the optimal transfer mechanism, to give the right incentives to the state, put the no repudiation constraint bind for some realizations of the level of locally collected taxes. When deciding what kind of contract to negotiate at date 0, the state could optimally choose to get no insurance and solve the autarky problem. These situation are more likely to emerge when the state starts negotiating with a low realization of its resources and negotiation costs are present, but independently of nationally collected taxes.

We should mention that this was, in some sense, a partial equilibrium approach since we have considered only one state negotiating with the central government. One could think that "the state" is actually representing a set of states. However, the natural extension would be to include any number of states; the easy conjecture is nevertheless that basic results would still hold whenever the optimal transfer mechanism ...nds situations where the no repudiation constraint is binding. It is clear that all these results crucially depend upon the assumption that is "technologically" given: if they could negotiate with respect to this, then the analysis should be very different.

As already mentioned, we were assuming away the possibility that the CG could repudiate any contract. However, this is a really strong assumption: we need to suppose that the federal system can make the CG commit to a given contract and cannot completely do this with a state. It would be interesting to analyze the possibility of having incomplete commitment on this side too.

Finally, we were assuming that the state cannot negotiate any other kind of debt. This is not a trivial assumption, in particular, when externalities with respect to risk premium paid for any agent in the economy are considered (assuming that the elasticity for these funds are finite). It could be actually the case that borrowing constraints to the states are Pareto improving.

REFERENCES

1. asdrubali, sorensen and yosha (1996): "Channels of Interstate Risk Sharing: United States 1963-1990", Quarterly Journal of Economics, November, 1081-1110.

2. atkeson, a (1991): "International Lending with Moral Hazard and Risk of Repudiation", Econometrica, 9, 1069-1089.

3. billingsley, p (1995): Probability and Measure, New York: John Wiley & Sons, Inc. Third Edition.

4. del negro, marco (1998): "Risk Sharing Across US States and European Coun-

tries: An Empirical Puzzle", mimeo, ITAM, Mexico.

5. espino, e (1999): "On E¢cient Allocations in Economies with Asymmetric Information and Incomplete Degree of Enforceability", mimeo, Cornell University, Department of Economics.

6. persson and tabellini (1996): "Federal Fiscal Constitutions: Risk Sharing and Moral Hazard", Econometrica, 64, 623-646.

7. persson and tabellini (1996): "Federal Fiscal Constitutions: Risk Sharing and Redistribution", Journal of Political Economy, 104, 979-1009.

8. sanguinetti and tomassi (1999): "Intergovernmental Transfers and Fiscal Behavior: Insurance Versus Aggregate Discipline", Working Paper, Torcuato Di Tella University, Buenos Aires.

9. stokey and lucas with prescott (1989): Recursive Methods in Economics Dynamics, Cambridge: Harvard University Press.